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Self-affine roughness influence on the pull-in voltage in capacitive electromechanical devices

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In this work we investigate the influence of self-affine roughness parameters on the pull-in voltage in capacitive microelectromechanical devices. The capacitor plate roughness is considered as self-affine type, which is described by the roughness amplitude w, the lateral correlation length ξ, and the roughness exponent H. By comparing the influence of the three parameters, we confirm that not only the long-wavelength roughness parameters w and ξ, but also the short-wavelength fine roughness details, as described by the roughness exponent H, play a major role. Therefore, the proper characterization of the involved surface roughness and its evolution at all relevant length scales are necessary to gauge properly the performance of associated devices. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2005376]

I. INTRODUCTION

Microelectromechanical (MEM) devices are no longer niche applications but they represent potential system for present and future technologies. Electrically supported microstructures become unstable under a nonlinear electrostatic force beyond an applied voltage, which is called the pull-in voltage.[1–4] A capacitive microelectromechanical system (MEMS) has been proposed for use as a dc voltage transfer standard in metrology.[1–5] In any case, the *pull-in* voltage depends only on a spring constant and geometrical properties. The moving plate of a parallel-plate capacitor, which is suspended by single-crystal silicon springs, can be very stable in comparison to Zener diodes. However, the latter have an inherent $1/f$ noise and long-term stability limitations.[6]

The capacitive devices can be designed to operate at any voltage between, e.g., one and several hundred volts. Device stability can be achieved with the use of feedback electronics, so that the noise of the dc voltage reference is mostly from mechanical sources.[5] When a dc voltage V_p is applied across the capacitor moving plate an electrostatic force,[1]

$$F_{\mathrm{el}} = \frac{C_{\mathrm{flat}} V_P^2 d}{2(d-x)^2}, \tag{1}$$

is generated, where x is the deflection, d is the gap at zero deflection, A is the capacitor plate area, $C_{\mathrm{flat}}=\varepsilon A/d$ is the capacitance for $x=0$ (zero deflection), and ε is the dielectric constant (Fig. 1). The mechanical force $F_{\mathrm{mech}}=-kx$, with k a spring constant, opposes the electrostatic force F_{el} to restore the position. The maximum voltage that can be applied to the device or the pull-in voltage is given by.[1]

$$V_{\mathrm{pi},f} = \sqrt{\frac{8kd^2}{27C_{\mathrm{flat}}}}, \tag{2}$$

assuming the capacitor plates are flat. In reality, however, surfaces are never perfectly smooth on nanometer length scales and usually roughness develops through the growth process of metal-deposited films (e.g., kinetic roughening and stress release) that will serve as electrodes or capacitor plates.

In this work we shall consider the influence of roughness for the case of random self-affine rough plates. In this case, besides of the rms roughness amplitude w and the lateral correlation length ξ, the short-wavelength roughness could

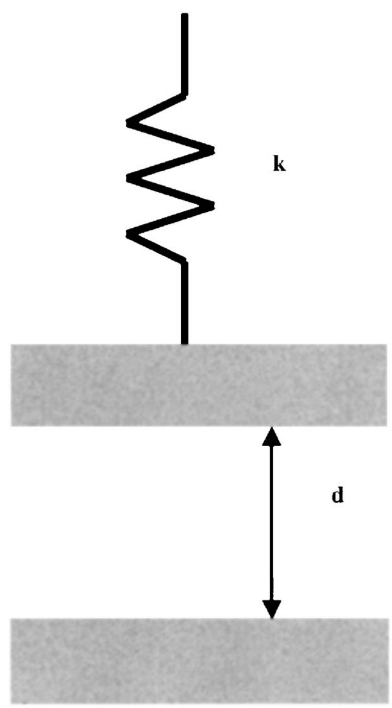

FIG. 1. Illustrative schematic of the capacitor-spring system.

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also play a critical role. The latter is characterized by a roughness exponent $H(0<H<1)$, which is a measure of the degree of surface irregularity.[7,8] Notably, this type of morphology occurs during metal film growth, which is necessary for various systems designed to operate with metal electrodes as capacitors in MEMS.[9–11]

II. BRIEF THEORY OF CAPACITORS WITH ROUGH PLATES

Consider a parallel-plate capacitor with only one rough electrode surface and the other one smooth. In order to calculate the electrical capacitance, one needs to solve the Laplace equation for the electrostatic potential Φ between the capacitor planes $\nabla^2\Phi(x,y,z)=0$, obeying the boundary conditions $\Phi(x,y,z=0)=0$, and $\Phi(x,y,z_b)=V_p$, where $z_b=d+h(\boldsymbol{\rho})$ [with $h(\boldsymbol{\rho})$ the roughness fluctuations and $\boldsymbol{\rho}=(x,y)$ the two-dimensional position vector] is the rough electrode surface held at potential V_p.[12] Perturbation theory up to second order yields the electric field[12]

$$\begin{aligned} E = & -\frac{V_p}{d}\hat{z} + \frac{V_p}{d}\int d\mathbf{k} f_1 k h(\mathbf{k}) e^{-I\mathbf{k}\cdot\boldsymbol{\rho}}\hat{z} \\ & -\frac{V_p}{d}\int d\mathbf{k}\int d\mathbf{k}' f_2 k' k h(\mathbf{k}') h(\mathbf{k}-\mathbf{k}') e^{-i\mathbf{k}\cdot\boldsymbol{\rho}}\hat{z} \\ & -\frac{V_p}{d}\int d\mathbf{k} f_3 k h(\mathbf{k}) e^{-i\mathbf{k}\cdot\boldsymbol{\rho}} i\mathbf{k}\frac{V_p}{d}\int d\mathbf{k} \\ & \times\int d\mathbf{k}' f_4 k' h(\mathbf{k}') h(\mathbf{k}-\mathbf{k}') e^{-i\mathbf{k}\cdot\boldsymbol{\rho}} i\mathbf{k}, \end{aligned} \tag{3}$$

where $f_1=\cosh(kz)/\sinh(kd)$, $f_2=[\cosh(k'd)\cosh(kz)/\sinh(k'd)\sinh(kd)]$, $f_3=\sinh(kz)/\sinh(kd)$, and $f_4=[\cosh(k'd)\sinh(kz)/\sinh(k'd)\sinh(kd)]$. The capacitance calculation is as follows: the surface charge density σ on the rough capacitor plate is given by $\varsigma=\varepsilon\mathbf{E}\cdot\hat{n}$ with $\hat{n}=(\nabla h-\hat{z})/[1+(\nabla h)^2]^{1/2}$ being the unit vector normal to the rough surface plate at $z=d+h(\boldsymbol{\rho})$. Upon ensemble average over possible roughness configurations assuming statistically stationary surfaces up to second order or $\langle h(\mathbf{k})h(\mathbf{k}')\rangle=[(2\pi)^4/A]\langle|h(\mathbf{k})|^2\rangle\delta(\mathbf{k}+\mathbf{k}')$, as well as assuming weak surface roughness or $|\nabla h|<1$, the average electric capacitance $C=\langle Q\rangle/V=\int\langle\sigma\rangle ds/V$ is given by[12]

$$\begin{aligned} C_r = C_{\text{flat}}\Bigg[& 1 + \frac{2(2\pi)^4}{A}\int_{0<k<Q_c} k^2\langle|h(\mathbf{k})|^2\rangle d\mathbf{k} \\ & + \frac{(2\pi)^4}{Ad}\int_{0<k<Q_c}\frac{\cosh(kd)}{\sinh(kd)} k\langle|h(\mathbf{k})|^2\rangle d\mathbf{k}\Bigg], \end{aligned} \tag{4}$$

where $Q_c=\pi/a_o$ with a_o of the order of atomic dimensions. Furthermore, calculation of the roughness influence in Eq. (4) requires knowledge of the roughness spectrum $\langle|h(\mathbf{k})|^2\rangle$.

III. RESULTS AND DISCUSSION

Indeed, a wide variety of surfaces and interfaces appearing in various physical systems (i.e., films grown under nonequilibrium conditions) possess self-affine roughness.[7] In this case, the roughness spectrum $\langle|h(\mathbf{k})|^2\rangle$ shows the power-law scaling $\langle|h(\mathbf{k})|^2\rangle\propto k^{-2-2H}$ if $k\xi\gg 1$, and $\langle|h(\mathbf{k})|^2\rangle\propto$ const if $k\xi\ll 1$.[7] This scaling is satisfied by the analytic model[8]

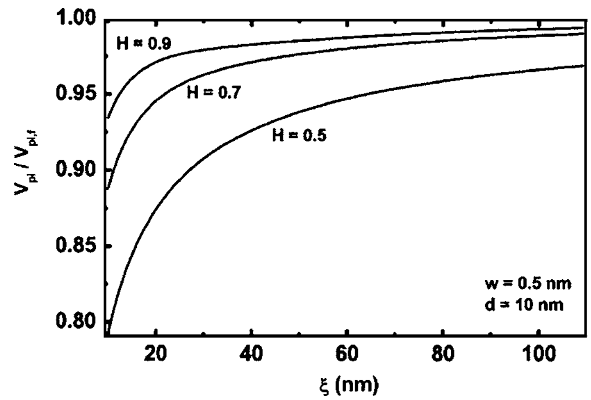

FIG. 2. Pull-in voltage ratio $V_{\text{pi}r}/V_{\text{pi}f}$ vs correlation length ξ for various roughness exponents H.

$$\langle|h(\mathbf{k})|^2\rangle = \frac{A}{(2\pi)^5}\frac{w^2\xi^2}{(1+ak^2\xi^2)^{1+H}}, \tag{5}$$

with $a=1/2H[1-(1+aQ_c^2\xi^2)^{-H}](0<H<1)$, and $a=1/2\ln(1+aQ_c^2\xi^2)(H=0)$.[8] Note that small values of $H(\sim 0)$ characterize extremely jagged or irregular surfaces, while larger values of $H(\sim 1)$ surfaces with smooth hills and valleys.[7]

Substitution of Eq. (5) in Eq. (4) yields for the capacitance ratio

$$\begin{aligned} \frac{C_r}{C_{\text{flat}}} = & 1 + 2\frac{w^2}{2a\xi^2}\left\{\frac{1}{1-H}[(1+aQ_c^2\xi^2)^{1-H}-1]-2a\right\} \\ & + \frac{1}{d}\int_0^{Q_c}\frac{\cosh(kd)}{\sinh(kd)}k^2\frac{w^2\xi^2}{(1+ak^2\xi^2)^{1+H}}dk, \end{aligned} \tag{6}$$

where upon substitution for the pull-in voltage we obtain,

$$V_{\text{pi},r} = V_{\text{pi},f}\left[\frac{C_r}{C_{\text{flat}}}\right]^{-1/2}. \tag{7}$$

Since $\langle|h(k)|^2\rangle\propto w^2$, the dependence of the pull-in voltage on the roughness amplitude is rather simple, while any more complex dependence will arise from the roughness parameters H and ξ.

Figure 2 shows calculations of the pull-in ratio as a function of the lateral correlation length ξ for various roughness exponents H. The variation as a function of the correlation length ξ is more drastic for smaller roughness exponents H. This is indicative of the fact that short-wavelength roughness, which is characterized by the roughness exponent H, can strongly alter the influence of long-wavelength roughness as it is expressed, e.g., by the lateral correlation length ξ. Indeed, as the roughness exponent H increases from 0 to 1, the pull-in voltage is strongly influenced at a comparable or even larger magnitude than that of the lateral correlation length ξ.

Figure 3 shows the direct variation as a function of the roughness exponent H for different plate separations d. It is clearly illustrated that the direct influence of short-wavelength or power-law roughness which is becoming in-

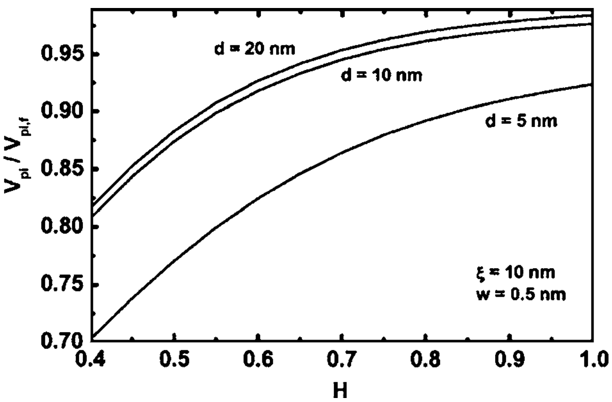

FIG. 3. Pull-in voltage ratio $V_{\mathrm{pi}r}/V_{\mathrm{pi}f}$ vs roughness exponent H for various plate separations d.

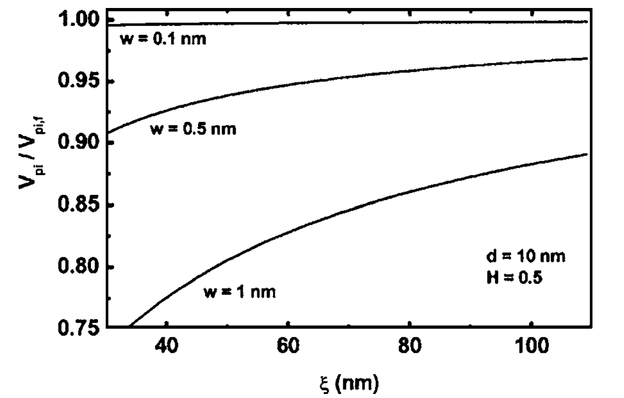

FIG. 5. Pull-in voltage ratio $V_{\mathrm{pi}r}/V_{\mathrm{pi}f}$ vs correlation length ξ for various roughness amplitudes w.

dependent on plate separation for distances $d>d_o$ (see also Fig. 4), where d_o is given by the complex relation $\int_{0<k<k_c} k^2\langle|h(\mathbf{k})|^2\rangle d\mathbf{k}=(1/d_o)\int_{0<k<k_c}[\cosh(kd_o)/\sinh(kd_o)]k\langle|h(\mathbf{k})|^2\rangle d\mathbf{k}$. The latter yields an effective value for $d_o\sim 10$ nm. In this case the pull-in voltage obtains the simpler expression since the second term on the right-hand side of Eq. (6) ceases to contribute (see also Appendix),

$$V_{\mathrm{pi},r} \approx V_{\mathrm{pi},f}\left(1+\frac{w^2}{a\xi^2}\left\{\frac{1}{1-H}[(1+aQ_c^2\xi^2)^{1-H}-1]-2a\right\}\right)^{-1/2}. \tag{8}$$

Figure 4 shows the influence of the plate separation with evolving correlation length ξ. In order to obtain the correct limiting form for $H=1$, one has to employ the identity $\ln(u)=\lim_{m\to 0}(1/m)(u^m-1)$. Therefore, from Eq. (8) we have for $H=1$ the simple form $V_{\mathrm{pi},r}\approx V_{\mathrm{pi},f}\{1+(w^2/a\xi^2)[\ln(1+aQ_c^2\xi^2)-2a]\}^{-1/2}$.

In order to gauge the influence of all roughness parameters, Fig. 5 shows also the direct dependence of the pull-in voltage ratio on the roughness amplitude w. Indeed, more pronounced variation with increasing correlation length ξ takes place for larger values of the roughness amplitude w. At any rate, if we compare the influence of all roughness parameters from Figs. 2, 3, and 5 we can infer that not only the long-wavelength roughness parameters w and ξ, but also the short-wavelength fine roughness details, as they are described by the roughness exponent H, play a major role on the magnitude of the pull-in voltage.

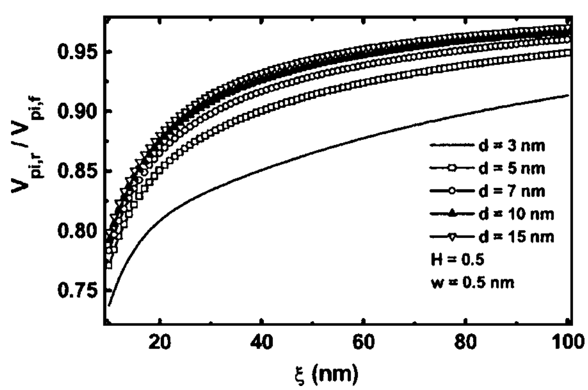

FIG. 4. Pull-in voltage ratio $V_{\mathrm{pi}r}/V_{\mathrm{pi}f}$ vs correlation length ξ for various capacitor plate separations d.

So far we neglected any influence by the evolution of growth front. However, upon metal film deposition which is commonly encountered in MEMS, surface roughness can evolve with deposition time τ (Refs. 7 and 8) as, for example, $w\tau^{\beta}$ (with $\beta<1$ the so-called growth exponent) and $\xi\tau^{1/z}$ (with z the dynamic exponent). Indeed, if we consider for simplicity the case of relatively significant separation so that Eq. (8) to apply, if the scaling exponents β, z, and H satisfy the relation $z=H/\beta$ (*normal self-affine growth*),[7,13,14] then the local surface slope is time invariant of the growth process. As a result it will contribute a constant factor on the pull-in voltage reduction since $V_{r,\mathrm{pi}}\approx V_{\mathrm{flat,pi}}\{1-[1/2(1-H)Q_c^{2(H-1)}a^H](w^2/\xi^{2H})\}$ and $w^2/\xi^{2H}\approx$ const [assuming $H<1$ and $Q_c\xi\gg 1$, see also Eq. (A2) in Appendix]. If, however, the condition $z=H/\beta$ is not fulfilled (*anomalous self-affine growth*[13]) then the local slope evolves with the film deposition and that will lead to changes of the pull-in voltage. Moreover, the local slope can also increase logarithmically with deposition time as $\rho_{\mathrm{rms}}\sim\sqrt{\ln(\tau)}$ (where due to insufficient surface diffusion of deposited metal atoms a groove instability develops leading to anomalous scaling)[13] that will also lead to the pull-in voltage dependence on deposition time of the form $V_{r,\mathrm{pi}}\approx V_{\mathrm{flat,pi}}[1-C\ln(\tau)]$.

IV. CONCLUSIONS

In summary we have examine the influence of characteristic self-affine roughness parameters on the pull-in voltage behavior. If we compare the influence of all roughness parameters we can infer that not only the long-wavelength roughness parameters w and ξ, but also the short-wavelength roughness fine roughness details, as they are described by the roughness exponent H, play a major role. Therefore, the proper characterization of the involved surface roughness in MEMS is necessary in order to gauge properly their performance. Note also that upon metal film deposition, which is commonly encountered, the in MEMS surface roughness can evolve,[7,8,13] which is also a factor that has to be taken into account.

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APPENDIX

Equation (4) is valid for weak roughness or small average local surface slopes. The latter is given by $\rho_{\rm rms} = \sqrt{\langle |\nabla h|^2 \rangle} \ll 1$,[14]

$$\rho_{\rm rms} = \left\{ \frac{(2\pi)^4}{A} \int_{0<k<Q_c} k^2 \langle |h(\mathbf{k})|^2 \rangle d\mathbf{k} \right\}^{1/2}. \tag{A1}$$

Therefore, under the condition of weak slopes Eq. (8) obtains the form (to first order)

$$V_{r,\rm pi} V_{\rm flat,pi} \left(1 - \frac{w^2}{2a\xi^2} \left\{ \frac{1}{1-H} [(1 + aQ_c^2 \xi^2)^{1-H} - 1] - 2a \right\} \right). \tag{A2}$$

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